Lecture 3: Information in Sequential Screening

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Motivation

A seller wants to sell an object to a prospective buyer(s).

- Buyer has imperfect private information θ about value v.
- The seller controls additional signal about v.
- The seller can partially or fully disclose her signal to buyer.

Disclosure is *private*:

- Seller cannot observe the realization of the disclosed signal, or
- seller observes signal realization but does not know how it enters buyer's utility function.

Research question:

- What is the *jointly* optimal selling mechanism and disclosure policy?

Sequential Learning and Information Control

Buyers often receive information sequentially:

- buyers start with some initial incomplete information
- they receive additional information later
- airline tickets, hotel booking, new products, business assets, fine art and estate ...

Sellers often have substantial control over information:

- supply production information (Lewis and Sappington, 1994, Johnson and Myatt, 2006)
- control access to information in indicative bidding (Ye, 2007)
- control how buyers learn by restricting the number and nature of tests they can carry out

Lecture Plan

Will first cover the sequential screening model of Courty and Li (2000).

Will then cover the full disclosure result of Esö and Szentes (2007).

– This results depends on a common orthogonalization trick in dynamic mechanism design.

Then, will discuss how information disclosure can be profitable if seller controls correlated shocks (Li and Shi 2015).

– At a technical level, highlights the limitation of orthogonal decomposition approach to information disclosure.

Courty and Li (2000): Sequential Screening

Review of Economic Studies

Sequential Screening Example

- Consider an example of airplane ticket pricing.
- Seller has a cost of 1 per seat.
- 1/3 are leisure travelers whose valuation is $\mathbb{U}[1,2]$
- 2/3 are business travelers whose valuation is $\mathbb{U}[0,1] \cup [2,3]$.
  - Business travelers face greater valuation uncertainty.
- Once travelers have privately learned their valuations, the value distribution is $\mathbb{U}[0,3]$.
- Monopoly price is 2 with expected profit of 1/3.
  - All leisure travelers and half of business travelers are excluded.

Sequential Screening Example

- Suppose instead that the seller offers two contracts before the travelers learn their valuation.
- These contracts consist of an advance payment and refund.
  - The first has an advance payment of 1.5 and no refund.
  - The other has an advance payment of 1.75 and a partial refund of 1.
- Leisure travelers strictly prefer the contract with no refund.
- Business travelers are indifferent between the two contracts so assume they choose refund contract.
- The monopolist separates the two types and earns an expected profit of 2/3.
  - Double the monopoly profits after values are learned.

Sequential Screening

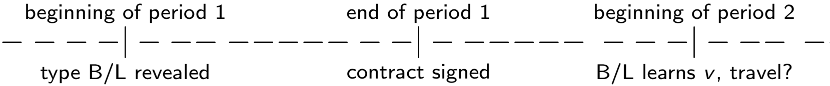
- I will analyze both the discrete and continuous setting.
- The discrete setting is straightforward
- The continuous setting is trickier.
  - Necessary conditions for IC is straightforward.
  - Sufficient conditions for IC is tricky: easier for FSD, harder for MPS.

Discrete Model

- A monopolist airline with unit cost c faces two types of travelers, $\theta \in \{B, L\}$
  - Proportions of B and L travelers: f_B and f_L
  - Type B and L travelers value the ticket v_B and v_L
  - Valuation distributions: $v_B \sim G_B$ and $v_L \sim G_L$
- Both seller and travelers are risk neutral, and do not discount
- Multi-dimensional mechanism design problem
  - But consumers are screened twice (sequentially), instead of just once
  - Can be modeled as a static problem in the first period, where travelers choose a package of delivery probabilities and transfer payments contingent on realization of valuations
  - For discrete model, we use indirect mechanisms: advance payment and refund

Timing of the Game

- period 1
  - the traveller first privately learns his type θ
  - the seller and the traveller contract at the end of period 1
- period 2
  - the traveller privately learns his actual valuation v for the ticket, and then decide whether to travel.



Ranking Distributions

- Consider the following two ways in which B and L are ordered.
- First-order stochastic dominance (FSD)
  - G_B dominates G_L by FSD if $G_B(v) \leq G_L(v)$ for all $v \in [\underline{v}, \overline{v}]$
  - business travellers stochastically have higher valuations
- Second-order stochastic dominance (MPS)
  - G_B dominates G_L by MPS if they have the same mean and $\int_{\underline{v}}^{v} [G_B(s) - G_L(s)]\, ds \geq 0$ for all $v \in [\underline{v}, \overline{v}]$
  - business travellers stochastically have higher uncertainty

Refund Contract

- refund contract (a, k)
  - an advance payment a at the end of period 1
  - a refund k that can be claimed at the end of period 2 after the traveller learns v
- under refund contract (a, k)
  - traveller will travel only if $v \geq k$
  - type $\theta \in \{B, L\}$ traveller's expected payoff at the end of period 1:

$$-a + kG_\theta(k) + \int_k^{\overline{v}} v dG_\theta(v)$$

Seller's Optimization Problem

The seller offers a menu of contracts $\{(a_B, k_B), (a_L, k_L)\}$ to maximize her revenue:

$$f_B [a_B - k_B G_B(k_B) - c(1 - G_B(k_B))] + f_L [a_L - k_L G_L(k_L) - c(1 - G_L(k_L))]$$

subject to

$$IR_B \quad : \quad -a_B + k_B G_B(k_B) + \int_{k_B}^{\overline{v}} v dG_B(v) \geq 0$$

$$IR_L \quad : \quad -a_L + k_L G_L(k_L) + \int_{k_L}^{\overline{v}} v dG_L(v) \geq 0$$

$$IC_B \quad : \quad -a_B + k_B G_B(k_B) + \int_{k_B}^{\overline{v}} v dG_B(v) \geq -a_L + k_L G_B(k_L) + \int_{k_L}^{\overline{v}} v dG_B(v)$$

$$IC_L \quad : \quad -a_L + k_L G_L(k_L) + \int_{k_L}^{\overline{v}} v dG_L(v) \geq -a_B + k_B G_L(k_L) + \int_{k_B}^{\overline{v}} v dG_L(v)$$

Reformulation

- under either FSD or MPS, IR_L and IC_B implies IR_B
- constraints IR_L and IC_B are binding, while IC_L is redundant
- seller chooses $\{(a_B, k_B), (a_L, k_L)\}$ to maximize her revenue

$$f_B [a_B - k_B G_B (k_B) - c (1 - G_B (k_B))] + f_L [a_L - k_L G_L (k_L) - c (1 - G_L (k_L))]$$

subject to IR_L and IC_B.

Reformulation

Using IC_B and IR_L, write the advance payments a as a function of the refund k.

Plug back into the seller's problem to get

$$\max_{k_B,,k_L} \left\{ \begin{array}{c} \underbrace{f_B \int_{k_B}^{\overline{v}} (v-c)\, dG_B(v)}_{\widehat{S}(k_B)\text{: suplus from type B}} + \\ \underbrace{f_L \int_{k_L}^{\overline{v}} (v-c)\, dG_L(v)}_{S(k_L)\text{: surplus from type L}} - \underbrace{f_B \int_{k_L}^{\overline{v}} \left[G_L(v) - G_B(v)\right] dv}_{R(k_L)\text{: rent for type B}} \end{array} \right\}$$

Optimal refund:

$$k_B = c \text{ and } k_L \in \arg\max_k \left[f_L S(k) - f_B R(k)\right].$$

Optimal Refund Contract under FSD

- suppose G_B dominates G_L in first-order stochastic dominance (FSD)
  - optimal contract: $k_L \geq c$
  - excessive refund or under-consumption for the L type
- intuition
  - under FSD, the rent $R(k_L)$ for the B type is decreasing in k_L
  - surplus $S(k_L)$ is increasing for any $k_L < c$, so an increase in k_L increases surplus and reduces rent

Optimal Refund Contract under MPS

Single mean-preserving spread (MPS): G_B crosses G_L only once and from above at z, and g_B and g_L are symmetric around z:

$$\begin{array}{lll} G_L(v) - G_B(v) < 0 & \text{if} & v < z \\ G_L(v) - G_B(v) > 0 & \text{if} & v > z \end{array}.$$

- Eg: G_B, G_L are normal with same mean, different variance.
- if $c < z$, subsidize the low type, i.e., insufficient refund $k_L < c$ or selling the ticket below c.
- if $c > z$, ration the low type, i.e., excess refund $k_L > c$ or selling the ticket above c.

Intuition:

- if $c < z$, rationing is costly because it prevents profitable trade.
- if $c > z$, subsidy is costly because it leads to inefficient. trade

Continuous Model

- ex ante types $\theta \sim F(\cdot)$ with a density function $f(\theta)$
  - each type θ represents a distribution of valuations with pdf $g(v|\theta)$ and cdf $G(v|\theta)$.
  - θ could be information about expected valuation (FSD) or the degree of valuation uncertainty (MPS).
- distributions $g(v|\theta)$ have the same support for all θ
  - by revelation principle, focus on the direct revelation mechanism $\{x(\theta, v), t(\theta, v)\}$
  - allocation rule $x(\theta, v)$ and payment rule $t(\theta, v)$ given the report (θ, v)

Seller's Optimization Problem

The seller's maximization problem is given by

$$\max_{x(\theta,v),t(\theta,v)} \int_{\theta} \int_{v} \left[t\left(\theta,v\right)-x\left(\theta,v\right)c\right] g\left(v|\theta\right) f\left(\theta\right) dv d\theta$$

subject to

$$\begin{aligned} IC_2 &: \quad v \in \arg\max_{v'} x\left(\theta,v'\right)v - t\left(\theta,v'\right) \quad \forall\theta,\forall v \\ IC_1 &: \quad \theta \in \arg\max_{\theta'} \int_{v} \left[x\left(\theta',v\right)v - t\left(\theta',v\right)\right] g\left(v|\theta\right) dv \quad \forall\theta \\ IR &: \quad \int_{v} \left[x\left(\theta,v\right)v - t\left(\theta,v\right)\right] g\left(v|\theta\right) dv \geq 0 \quad \forall\theta \end{aligned}$$

Characterization of IC in Period 2

- consumer's ex post surplus after he truthfully reports θ and v:

$$u(\theta, v) = x(\theta, v)\, v - t(\theta, v)$$

- expected surplus of a consumer of type θ by reporting truthfully:

$$U(\theta, \theta) = \int_v u(\theta, v)\, g(v|\theta)\, dv$$

- second period IC constraints are satisfied if and only if
  - (M) $x(\theta, v)$ is nondecreasing in v.
  - (FOC) $u(\theta, v) = u(\theta, \underline{v}) + \int_{\underline{v}}^{v} x(\theta, s)\, ds$.

IC in Period 1

- rewrite $U(\theta)$ as

$$
\begin{aligned}
U(\theta) &= \max_{\theta'} \int_v u(\theta', v)\, g(v|\theta)\, dv \\
&= \max_{\theta'} \int_v \left[u(\theta', \underline{v}) + \int_{\underline{v}}^{v} x(\theta', s)\, ds \right] g(v|\theta)\, dv \\
&= \max_{\theta'} \left\{ u(\theta', \underline{v}) + \int_{\underline{v}}^{\overline{v}} [1 - G(v|\theta)]\, x(\theta', v)\, dv \right\}
\end{aligned}
$$

- we would like to use FOA to localize the first period IC constraints
  - but local first-order condition and monotonicity are not sufficient

Necessary Conditions for IC in Period 1

- first period IC constraints imply that
  - (M) $\int_{\underline{v}}^{\overline{v}} \left[G\left(v|\theta'\right) - G\left(v|\theta\right) \right] \left[x\left(\theta, v\right) - x\left(\theta', v\right) \right] dv \geq 0.$
  - (FOC) $U\left(\theta\right) = U\left(\underline{\theta}\right) - \int_{\underline{\theta}}^{\theta} \left[\int_{\underline{v}}^{\overline{v}} \frac{\partial G(v|s)}{\partial s} x\left(s, v\right) dv \right] ds.$
- (M) and (FOC) are necessary but not sufficient for IC_1

Seller's Relaxed Program

- use FOA to obtain a "relaxed" problem with ICs replaced by FOCs.
- seller's revenue is rewritten as

$$\int_{\underline{\theta}}^{\overline{\theta}} \int_{\underline{v}}^{\overline{v}} \left[t(\theta, v) - x(\theta, v)\, c\right] g(v|\theta)\, f(\theta)\, dv d\theta$$

$$= \int_{\underline{\theta}}^{\overline{\theta}} \int_{\underline{v}}^{\overline{v}} \left[v - c + \frac{1 - F(\theta)}{f(\theta)} \frac{\frac{\partial G(v|\theta)}{\partial \theta}}{g(v|\theta)}\right] x(\theta, v)\, g(v|\theta)\, f(\theta)\, dv d\theta$$

$$- U(\underline{\theta})$$

using integration by parts

Virtual Value Function

- virtual surplus function $J(\theta, v)$ is given by

$$J(\theta, v) = v - c + \frac{1 - F(\theta)}{f(\theta)} \frac{\frac{\partial G(v|\theta)}{\partial \theta}}{g(v|\theta)}.$$

  - informativeness measure: $\frac{\partial G(v|\theta)}{\partial \theta} / g(v|\theta)$
  - it represents the informativeness of the first-period type on second-period valuations.

- solution to the relaxed problem with monotone J is

$$x(\theta, v) = \begin{cases} 1 & \text{if} \quad J(\theta, v) \geq 0 \\ 0 & \text{if} \quad J(\theta, v) < 0 \end{cases}.$$

- when is (FOC) also sufficient for the IC constraints in period 1?

Sufficient Conditions for IC1 under FSD

- strong monotonicity: $x(\theta, v)$ is nondecreasing in both arguments.
- sketch of proof:

$$\begin{aligned} U(\theta) &= U(\theta, \theta') + \int_{\underline{v}}^{\overline{v}} G(v|\theta)\left[x(\theta', v) - x(\theta, v)\right] dv \\ &+ \int_{\underline{v}}^{\overline{v}} \int_{\theta'}^{\theta} G(v|s) \frac{\partial x(s, v)}{\partial s} ds dv \end{aligned}$$

If $\theta > \theta'$, $G(v|s) \geq G(v|\theta)$ for $s \in [\theta', \theta]$, and

$$\int_{\underline{v}}^{\overline{v}} \int_{\theta'}^{\theta} G(v|s) \frac{\partial x(s, v)}{\partial s} ds dv \geq \int_{\underline{v}}^{\overline{v}} G(v|\theta) \int_{\theta'}^{\theta} \frac{\partial x(s, v)}{\partial s} ds dv$$

The case with $\theta < \theta'$ is similar.

Sufficient Conditions for IC1 under MPS

- harder to find sufficient conditions under MPS
- additional restriction on distributions
  - all distributions passing through a single point z
- additional constraints on the allocation rule $x(\theta, v)$
  - if $c < z$, $x(\theta, v)$ is nonincreasing in θ for all v and nondecreasing in v for all θ
  - if $c > z$, $x(\theta, v)$ is nondecreasing in both θ and v
- one more condition
  - if $c < z$: no under production
  - if $c > z$: no over production

FSD Parameterization

- AR(1) process: $v = \gamma\theta + (1-\gamma)\,\varepsilon_\theta$, where $\gamma \in (0,1)$
  - ε_θ is iid with density $h(\cdot)$ and distribution $H(\cdot)$.
  - informativeness measure:

$$\frac{\frac{\partial G(v|\theta)}{\partial\theta}}{g(v|\theta)} = \frac{h\left(\frac{v-\gamma\theta}{1-\gamma}\right)\left(-\frac{\gamma}{1-\gamma}\right)}{h\left(\frac{v-\gamma\theta}{1-\gamma}\right)\left(\frac{1}{1-\gamma}\right)} = -\gamma.$$

- virtual surplus function

$$J(\theta, v) = v - c + \frac{1-F(\theta)}{f(\theta)}\frac{\frac{\partial G(v|\theta)}{\partial\theta}}{g(v|\theta)} = v - c - \gamma\frac{1-F(\theta)}{f(\theta)}.$$

  monotone in both v and θ if F has increasing hazard rate
- solution $x(\theta, v)$ to the relaxed problem is monotone both in v and θ.

MPS Parameterization

- suppose $v = z + \theta\varepsilon_\theta$, where ε_θ is iid with zero mean, density $h(\cdot)$ and distribution $H(\cdot)$
- informativeness measure

$$\frac{\frac{\partial G(v|\theta)}{\partial\theta}}{g(v|\theta)} = \frac{h\left(\frac{v-z}{\theta}\right)\left(-\frac{v-z}{\theta^2}\right)}{h\left(\frac{v-z}{\theta}\right)\left(\frac{1}{\theta}\right)} = -\frac{v-z}{\theta}.$$

- virtual surplus function

$$J(\theta, v) = v - c + \frac{1-F(\theta)}{f(\theta)}\frac{\frac{\partial G(v|\theta)}{\partial\theta}}{g(v|\theta)} = v - c - (v-z)\frac{1-F(\theta)}{\theta f(\theta)}.$$

- if F has increasing hazard rate, solution $x(\theta, v)$ to the relaxed problem also solves the original problem

Esö and Szentes (2007): Optimal Information Disclosure in Auctions and the Handicap Auction

Review of Economic Studies

The Model

Single seller with a unit good to sell.

n buyers each with single unit demand.

Seller's valuation for the good is normalized to 0.

Her objective is to maximize expected revenue.

Each buyer's pay-off is the negative of his payment to the seller, plus, in case he wins, the value of the object.

Buyer's Valuation

Buyer i's true valuation for the object is v_i.

He private observes a noisy signal θ_i of v_i.

θ_i is drawn from F_i (commonly known).

- $f_i/(1 - F_i)$ is assumed to be nondecreasing.

In addition, the seller can disclose an additional noisy signal z_i of v_i to buyer i.

- The seller cannot observe this signal.
- She may also choose to partially reveal z_i.

z_i is allowed to be correlated with θ_i.

- However, (θ_i, z_i) is drawn independently across i.

Buyer's Valuation

Since buyer is risk neutral, it is without loss to assume

$$v_i = \mathbb{E}[v_i \mid \theta_i, z_i].$$

- After observing the signal, the buyer knows his posterior value v_i.

Assume v_i is increasing in z_i.

Valuation Distribution

$H_{i\theta_i}$ denotes the (twice continuously differentiable) distribution of v_i conditional on θ_i

Assumptions on $H_{i\theta_i}$:

1. $\frac{\partial H_{i\theta_i}}{\partial \theta_i} < 0$: $\theta_i > \hat{\theta}_i \implies$ $H_{i\theta_i}$ First Order Stochastically Dominates $H_{i\hat{\theta}_i}$,
2. $\frac{\partial H_{i\theta_i}(v_i)/\partial \theta_i}{h_{i\theta_i}(v_i)}$ is increasing in v_i,
3. $\frac{\partial H_{i\theta_i}(v_i)/\partial \theta_i}{h_{i\theta_i}(v_i)}$ is increasing in θ_i.

Interpretation: Substitutability in i's posterior valuation between θ_i and the part of z_i that is new to i.

Recap: Courty and Li (2000)

Here, the seller does not control information and $v_i = z_i$.

The seller screens by offering a menu of contracts with different allocations and prices.

- Can be thought of as a set of option/refund contracts.

Optimal allocation rule is given by the cutoff value that solves

$$v + \frac{\partial H_{i\theta_i}(v_i)/\partial \theta_i}{h_{i\theta_i}(v_i)} \frac{1 - F(\theta_i)}{f(\theta_i)} = 0.$$

As in Myerson, allocations pin down prices.

Orthogonalization

Suppose instead of z_i, the seller could disclose a new *independent* signal $s_i(z_i, \theta_i)$,

- s_i is strictly increasing in z_i, hence preserves information of z_i.
- Put differently, buyer's posterior valuation is same whether he observes z_i or $s_i(z_i, \theta_i)$.
- Recall, seller cannot observe z_i so does not observe s_i.

Orthogonalization: Proof

Lemma: (i) There exist functions u_i and s_i, such that $u_i(\theta_i, s_i(z_i, \theta_i)) := v_i$, such that u_i is strictly increasing, s_i is strictly increasing in z_i, and $s_i(z_i, \theta_i)$ is independent of θ_i.
(ii) All s_i's satisfying part (i) are positive monotonic transformations of each other.

Proof: Define $s_i(z_i, \theta_i) := H_{i\theta_i}(v_i)$, the percentile of the distribution of $v_i|\theta_i$.

$$\Pr\left(H_{i\theta_i}(v_i) \leq y\right) = \Pr\left(v_i \leq H_{i\theta_i}^{-1}(y)\right) = H_{i\theta_i}\left(H_{i\theta_i}^{-1}(y)\right) = y.$$

Note that s_i is uniform on $[0, 1]$ irrespective of θ_i and hence independent of θ_i.

Finally, define $u_i(\theta_i, s_i) := H_{i\theta_i}^{-1}(s_i)$.

Interpretation of Distributional Assumptions

Lemma:
(i) $\frac{\partial H_{i\theta_i}(v_i)/\partial\theta_i}{h_{i\theta_i}(v_i)}$ increasing in v_i implies $u_{i12} \leq 0$.
(ii) $\frac{\partial H_{i\theta_i}(v_i)/\partial\theta_i}{h_{i\theta_i}(v_i)}$ increasing in θ_i implies $\frac{u_{i11}}{u_{i1}} \leq \frac{u_{i12}}{u_{i2}}$.

Interpretation:

(i) The marginal impact of the s_i shock on i's valuation is non-increasing in his type θ_i.
(ii) An increase in i's type, holding the ex-post valuation constant, weakly decreases the marginal value of θ_i.

Benchmark: Seller Observes s_i

Suppose, the seller observes s_i.

- In this benchmark, the revenue must be weakly higher then any unobserved signal structure as this additional information can be ignored.

The seller's revenue can be written as a function of the allocation X_i:

$$\int_{\theta_i} \int_{s_i} \left(u_i(\theta_i, s_i) - \frac{1 - F(\theta_i)}{f(\theta_i)} u_{i1}(\theta_i, s_i) \right) X_i(\theta_i, s_i) dF(\mathbf{v}) dG(\mathbf{s})$$

Optimal allocation X_i^* assigns the good to the highest non-negative virtual value (follows from the same arguments as the Myerson auction).

Properties of Optimal Benchmark Allocation

Virtual Value: $u_i(\theta_i, s_i) - \frac{1-F(\theta_i)}{f(\theta_i)} u_{i1}(\theta_i, s_i)$

Lemma:

(i) X_i^* is continuous in both arguments.

(ii) X_i^* is weakly increasing in both arguments.

(iii) If $\theta_i > \hat{\theta}_i$, $s_i < \hat{s}_i$ and $u_i(\theta_i, s_i) = u_i(\hat{\theta}_i, \hat{s}_i)$, then $X_i^*(\theta_i, s_i) \geq X_i^*(\hat{\theta}_i, \hat{s}_i)$.

These properties imply that X_i^* can be implemented even when the seller does not observe s_i.

Consistent Deviations

Lemma:

In the second round of an IC two-stage mechanism, θ_i who reported $\hat{\theta}_i$ in the first round and has observed s_i will report $\hat{s}_i = \sigma_i(\theta_i, \hat{\theta}_i, s_i)$ such that

$$u_i(\theta_i, s_i) \equiv u_i(\theta_i, \sigma_i(\theta_i, \hat{\theta}_i, s_i))$$

Proof: If true value and signal were $\hat{\theta}_i$, $\hat{s}_i$, period 2 IC would imply

$$u_i(\hat{\theta}_i, \hat{s}_i)X_i^*(\hat{\theta}_i, \hat{s}_i) - T_i^*(\hat{\theta}_i, \hat{s}_i) \geq u_i(\hat{\theta}_i, \hat{s}_i)X_i^*(\hat{\theta}_i, s_i') - T_i^*(\hat{\theta}_i, s_i'),$$

for all s_i'.

This implies that

$$u_i(\theta_i, s_i)X_i^*(\hat{\theta}_i, \hat{s}_i) - T_i^*(\hat{\theta}_i, \hat{s}_i) \geq u_i(\theta_i, s_i)X_i^*(\hat{\theta}_i, s_i') - T_i^*(\hat{\theta}_i, s_i').$$

Main Result

Theorem:
The benchmark mechanism can be implemented by the seller even without observing the buyer's shock.

A key reason this works is: $\theta_i > \hat{\theta}_i$, and $u_i(\theta_i, s_i) = u_i(\hat{\theta}_i, \hat{s}_i)$ implies that $X_i^*(\hat{\theta}_i, \hat{s}_i) \leq X_i^*(\theta_i, s_i)$.

In words, θ_i and a given ex-post valuation wins the object more often than he does with $\hat{\theta}_i$, but the same ex post valuation

This provides the appropriate monotonicity which is required for period 1 IC.

Main Result: Intuition

Consider the case where $n = 1$.

The benchmark allocation can be implemented by an option contract: a type θ_i, gets a period 2 strike price $p_i(\theta_i) = u_i(\theta_i, \tilde{s}_i)$, where $\tilde{s}_i$ solves

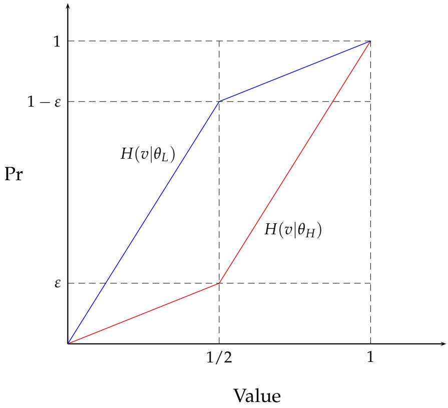
$$u_i(\theta_i, \tilde{s}_i) - \frac{1 - F(\theta_i)}{f(\theta_i)} u_{i1}(\theta_i, \tilde{s}_i) = 0.$$

By revenue equivalence, all implementations provide the same revenue (subject to binding IR of the lowest period 1 type).

In this implementation, s_i will be reported truthfully even if private information.

Li and Shi (2015): Discriminatory Information Disclosure
Working Paper

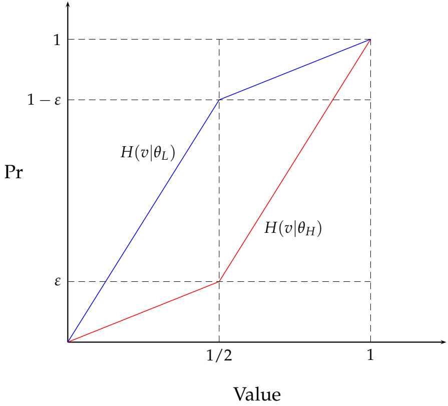
Binary Example



Buyer ex ante type $\theta \in \{\theta_H, \theta_L\}$, θ_H and θ_L equally likely.

$F(\cdot|\theta_H)$ and $F(\cdot|\theta_L)$ both piecewise uniform with $\varepsilon \in (0, 1/2)$:

Binary Example



Seller discloses, without observing, a noisy signal s of ω.

Seller's reservation value $c = 1/2$.

Total surplus $1/8$: $(1-\varepsilon)/4$ from θ_H, and $\varepsilon/4$ from θ_L.

Sequential Screening: $s = v$

- Buyer chooses between two option contracts before learning v:
  - high fee a^H for option of buying at efficient price $p^H = 1/2$.
  - low fee a^L for option of buying at high price $p^L > 1/2$.

- Optimal sequential screening
  - IC_H and IR_L bind: no rent for θ_L.
  - seller revenue = trading surplus − information rent for θ_H.
  - optimal $p^L = 1/2 + (1 - 2\varepsilon)/(2 - 2\varepsilon)$, balancing surplus and rent.
  - information rent for θ_H: $R_H = (1 - 2\varepsilon)(1 - p^L)^2 > 0$.
  - seller revenue: $\pi = \left(1 - \varepsilon\left(1 - 2\varepsilon\right) / \left(1 - \varepsilon\right)\right) / 8 < 1/8$.

Sequential Screening with Information Control

- Sequential screening with discriminatory disclosure
  - efficient contract for high type, hence full information.
  - inefficient contract for low type and partial information.
- Optimal menu with discriminatory disclosure
  - charge $a^H = (1 - \varepsilon)/4$ for full disclosure and set $p^H = 1/2$.
  - charge $a^L = 0$ for binary partition disclosure (whether $v \geq 1/2$), and set $p^L = 3/4$.
  - θ_H indifferent; θ_L strictly prefers binary partition.
- Extract entire trade surplus of $1/8$.

Model: Signals

- Consider a two-period sequential screening model
  - seller has a single object for sale, with reservation value $c \geq 0$.
  - both parties are risk-neutral, and do not discount.

- Buyer's underlying true valuation: $v \in \Omega = [\underline{v}, \overline{v}]$
  - $t = 1$: buyer privately observes signal $\theta \in \Theta$ about v (ex ante type).
  - primitive: $v|\theta \sim H(v|\theta)$, with CDF $F(\theta)$; for $\theta > \theta'$, $H(\cdot|\theta)$ first-order stochastic dominates $H(\cdot|\theta')$.

- Seller controls additional signal about v
  - $t = 2$: seller can release to buyer, without observing, a signal s.
  - given θ and s, buyer's posterior estimate of v is v.
  - θ and s are correlated.

Model: Signal Structure and Disclosure Policy

- Signal structure $\sigma \in \mathcal{S}$ is a joint distribution $H^{\sigma}(v, \theta, s)$ such that

$$\int_{s \in S} dH^{\sigma}(v, \theta, s) = H(v, \theta). \qquad \text{(consistency)}$$

  where S is the set of possible signal realizations.
- Disclosure policy, $\sigma(\theta) : \Theta \to \mathcal{S}$, assigns σ to reported type θ.
- Different classes of disclosure rules, with varying restrictions on the set of signal structures $\mathcal{S}$:
  - direct disclosure: signal does not depend on true type
  - general disclosure: no additional restriction other than consistency
  - classical sequential screening: $\mathcal{S} = \{\overline{\sigma}\}$, $\overline{\sigma}$: perfect signal structure
  - specific technologies: Gaussian, truth-or-noise...

Model: Direct Disclosure

- Direct disclosure:
  - signal structure $\sigma : \Omega \to \Delta S$, direct garbling of the perfect signal $\overline{\sigma}$
  - signal distribution under σ: $H^{\sigma}(s|v,\theta) = \Gamma^{\sigma}(s|v)$.
- Binary partition:
  - partition threshold $k \in (\underline{v}, \overline{v})$,
  - signal space $S = \{s_-, s_+\}$,
  - probability mass function $\gamma^{\sigma}(\cdot|v)$ corresponding to $\Gamma^{\sigma}(\cdot|v)$:

$$\gamma^{\sigma}(s|v) = \begin{cases} 1 & \text{if} \quad s = s_- \text{ and } v < k, \\ 1 & \text{if} \quad s = s_+ \text{ and } v \geq k, \\ 0 & \qquad\qquad \text{otherwise,} \end{cases}$$

  - probability of observing s_+ for a type-θ buyer under σ is $1 - H(k|\theta)$, which depends on the true type θ.

Model: Mechanism

Disclosure policy $\{\sigma(\theta)\}$ and direct mechanism $\{x(\theta, v), y(\theta, v)\}$:

- $\sigma(\theta)$ is the signal structure assigned for reported type θ.
- $x(\theta, v)$ is the trading probability conditional on buyer report (θ, v).
- $y(\theta, v)$ is the corresponding payment from buyer to seller.

Model: Timing

First period:

- v is realized, and buyer privately observes θ.
- seller commits to $\{\sigma(\theta)\}$ together with $\{x(\theta, v), y(\theta, v)\}$.
- buyer submits report $\tilde{\theta}$ about his type and $\sigma(\tilde{\theta})$ is implemented.

Second period:

- buyer observes additional signal $s_{\sigma(\tilde{\theta})}$ released by seller.
- buyer forms posterior estimate $v = \mathbb{E}\left[v \,\middle|\, \theta, s_{\sigma(\tilde{\theta})}\right]$ and reports $\tilde{v}$.
- contract $\{x(\tilde{\theta}, \tilde{v}), y(\tilde{\theta}, \tilde{v})\}$ is implemented.

Discrete Types

- Discrete ex ante type $\Theta = \{\theta_1, \ldots, \theta_n\}$, $f_i \equiv \Pr(\theta = \theta_i)$.
  - $H(v|\theta_i) \leq H(v|\theta_{i+1})$ for all i and all $v \in [\underline{v}, \overline{v}]$
- Restrict to deterministic selling mechanisms
  - menu of option contracts $\{a^i, p^i\}$
  - a^i is the non-refundable advance payment in period one
  - p^i is the corresponding strike price in period two
- Under full disclosure, a feasible contract $\{a^i, p^i\}$ satisfies:
  - IR_i: $-a^i + \int_{p^i}^{\overline{v}}(v - p^i)dH(v|\theta_i) \geq 0, \forall i$;
  - IC_{ij}: $-a^i + \int_{p^i}^{\overline{v}}(v - p^i)dH(v|\theta_i) \geq$
    $-a^j + \int_{p^j}^{\overline{v}}(v - p^j)dH(v|\theta_i), \forall i, j$.

Full Disclosure Not Optimal

- Proposition If ex ante types are ordered in FOSD, full disclosure ($\sigma^i = \overline{\sigma}$ for all i) is not optimal.

- Idea of proof:
  - take optimal contract $\left(a^i, p^i\right)$ under full disclosure;
  - for type $\theta_i \neq \theta_n$, keep $\sigma^i = \overline{\sigma}$ and strike price p^i;
  - for type θ_n, offer binary partition with cutoff p^n, raise strike price

    $$\widehat{p}^n = p^n + \delta, \text{ with } \delta \text{ small and strictly positive,}$$

    and reduce a^n to bind IR_1;
  - due to FOSD, price hike hurts deviating θ_i more than θ_n;
  - IC_{i1} are strictly slack, so we can uniformly raise a^i;
  - same allocation (hence surplus), but lower rent.

Discussion: Two Types

Monotone partitions need not be optimal

- The seller may want to pool high low values onto the same signal.

General disclosure may dominate direct disclosure.

- Types get no information if they misreport.

If $\int_{\underline{v}}^{\overline{v}} vdH(v|\theta_H) \leq \int_{c}^{\overline{v}} vdH(v|\theta_L)$, full surplus extraction is possible.

- Offer a binary monotone partition around c.
- Charge both types no upfront fee and a strike price $p_i = \int_c^{\bar{v}} vdH(v|\theta_i)$ for $i \in h, l$.

It is without loss to restrict to generalized monotone partitions.

Discussion: Contiuous Types

Direct disclosure policies are better than full disclosure.

Binary partitions are not optimal in general

- May be too informative for high type.